

Topics : Elasticity & Viscosity, Geometrical Optics, String Wave, Friction, Simple Harmonic Motion, Rigid Body Dynamics

Type of Questions

Single choice Objective ('-1' negative marking) Q.1 to Q.3

(12 marks, 12 min.)

M.M., Min.

[9, 9]

Subjective Questions ('-1' negative marking) Q.4 to Q.6

(4 marks, 5 min.)

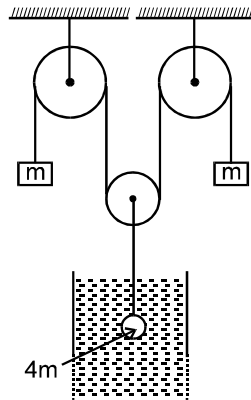
[12, 15]

Comprehension ('-1' negative marking) Q.7 to Q.9

(3 marks, 3 min.)

[9, 9]

1. A spherical ball of mass $4m$, density σ and radius r is attached to a pulley-mass system as shown in figure. The ball is released in a liquid of coefficient of viscosity η and density ρ ($< \frac{\sigma}{2}$). If the length of the liquid column is sufficiently long, the terminal velocity attained by the ball is given by (assume all pulleys to be massless and string as massless and inextensible) :



(A) $\frac{2 r^2 (2\sigma - \rho)g}{9 \eta}$

(B) $\frac{2 r^2 (\sigma - 2\rho)g}{9 \eta}$

(C) $\frac{2 r^2 (\sigma - 4\rho)g}{9 \eta}$

(D) $\frac{2 r^2 (\sigma - 3\rho)g}{9 \eta}$

2. Which of the following relations is correct for a spherical mirror if a point object is kept on the principal axis. ['P' is pole, 'C' is centre object is at point 'O', image is at point 'I']

(A) $\frac{OP}{OC} = \frac{IP}{IC}$

(B) $\frac{OP}{IC} = \frac{IP}{OC}$

(C) $\frac{PC}{PO} = \frac{PI}{PC}$

(D) $\frac{IO}{CP} = \frac{IP}{CO}$

3. A travelling wave $y = A \sin(kx - \omega t + \theta)$ passes from a heavier string to a lighter string. The reflected wave has amplitude $0.5 A$. The junction of the strings is at $x = 0$. The equation of the reflected wave is:

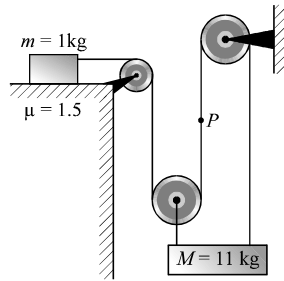
(A) $y' = 0.5 A \sin(kx + \omega t + \theta)$

(B) $y' = -0.5 A \sin(kx + \omega t + \theta)$

(C) $y' = -0.5 A \sin(\omega t - kx - \theta)$

(D) $y' = -0.5 A \sin(kx + \omega t - \theta)$

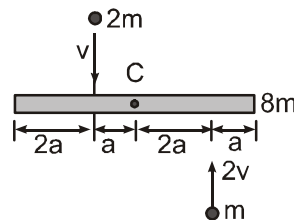
4. Figure shows an ideal pulley block of mass $m = 1 \text{ kg}$, resting on a rough ground with friction coefficient $\mu = 1.5$. Another block of mass $M = 11 \text{ kg}$ is hanging as shown. When system is released it is found that the magnitude of acceleration of point P on string is a . Find value of $4a$ in m/s^2 . (Use $g = 10 \text{ m/s}^2$)



5. A 900 kg elevator hangs by a steel cable for which the allowable stress is $1.15 \times 10^8 \text{ N/m}^2$. What is the minimum diameter required if the elevator accelerates upward at 1.5 m/s^2 . Take $g = 10 \text{ m/s}^2$ and leave your answer in terms of π .
6. A 40 kg mass, hanging at the end of a rope of length ℓ , oscillates in a vertical plane with an angular amplitude of θ_0 . What is the tension in the rope, when it makes an angle θ with the vertical? If the breaking strength of the rope is 80 kg f, what is the maximum angular amplitude θ with which the mass can oscillate without the rope breaking?

COMPREHENSION

A uniform bar of length $6a$ & mass $8m$ lies on a smooth horizontal table. Two point masses m & $2m$ moving in the same horizontal plane with speeds $2v$ and v respectively strike the bar as shown & stick to the bar after collision.



7. Velocity of the centre of mass of the system is
- (A) $\frac{v}{2}$ (B) v (C) $\frac{2v}{3}$ (D) Zero
8. Angular velocity of the rod about centre of mass of the system is
- (A) $\frac{v}{5a}$ (B) $\frac{v}{15a}$ (C) $\frac{v}{3a}$ (D) $\frac{v}{10a}$
9. Total kinetic energy of the system, just after the collision is
- (A) $\frac{3}{5} mv^2$ (B) $\frac{3}{25} mv^2$ (C) $\frac{3}{15} mv^2$ (D) $3 mv^2$



Answers Key

1. (B) 2. (A) 3. (D)

4. [13] 5. $\frac{6 \times 10^{-2}}{\sqrt{10} \pi} \text{ m}$

6. (a) $T = 40 (3 \cos \theta - 2 \cos \theta_0) \text{ kg f.}$ (b) $\theta_0 = 60^\circ$

7. (D) 8. (A) 9. (A)

Hints & Solutions

1. From the free body diagram of the sphere :

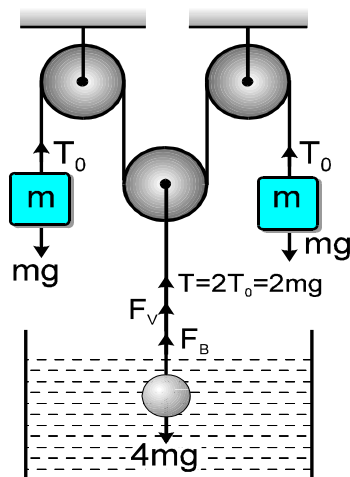
$$F_v = 4mg - 2mg - F_B$$

$$\Rightarrow F_v = 2mg - F_B$$

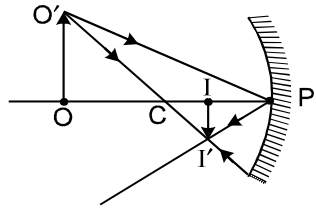
$$\Rightarrow 6\pi\eta r V = \frac{4}{3}\pi r^3 \left(\frac{\sigma}{2} - \rho \right) g$$

$$\text{(since } 4m = \frac{4}{3}\pi r^3 \times \sigma \text{)}$$

$$\Rightarrow V = \frac{2}{9} r^2 \frac{(\sigma - 2\rho)g}{\eta}$$



2. In the figure shown



$\therefore \triangle OO'P$ & $\triangle II'P$ are similar

$$\frac{OO'}{II'} = \frac{OP}{IP} \dots\dots\dots(1)$$

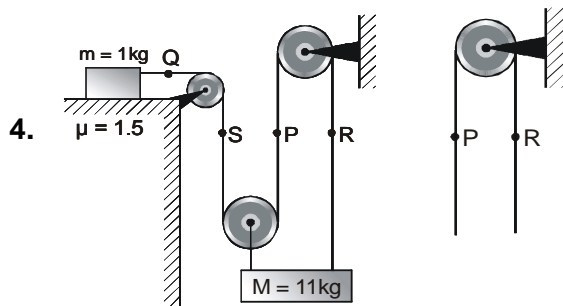
also $\therefore \triangle OO'C$ & $\triangle II'C$ are similar

$$\frac{OO'}{II'} = \frac{OC}{IC} \dots\dots\dots(2)$$

By equation (1) and (2)

$$\frac{OP}{IP} = \frac{OC}{IC} \Rightarrow \frac{OP}{OC} = \frac{IP}{IC} \text{ Ans.}$$

3. As wave has been reflected from a rarer medium, therefore there is no change in phase. Hence equation for the opposite direction can be written as
 $y = 0.5A \sin(-kx - \omega t + \theta)$
 $= -0.5A \sin(kx + \omega t - \theta)$



If the point P has an acceleration a upwards then the acceleration of point R will be a downwards.



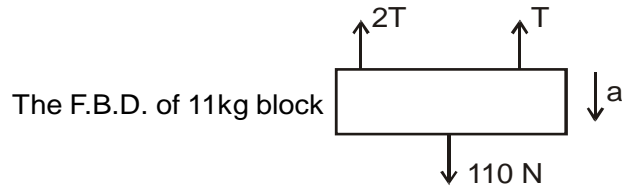
The point R has an acceleration a downwards so the block will also have an acceleration a downwards.



The point P has an acceleration a upwards, the block has an acceleration a downwards so the acceleration of S will be $3a$ downwards. (because

$$\frac{\bar{a}_S + \bar{a}_P}{2} = \bar{a}_{\text{block}}).$$

The point Q will also have an acceleration $3a$ towards right.



Using FBD of 11 kg block, which will have acceleration a downwards.

$$110 - 3T = 11a \dots\dots (1) \text{ (in downwards direction)}$$

For 1 kg block, which will have acceleration $3a$,

$$T - 15 = 3a \text{ (in horizontal direction)}$$

$$\text{or } 3T - 45 = 9a \dots\dots\dots (2)$$

on adding equation (1) & (2) we get

$$20a = 65 \Rightarrow 4a = 13 \text{ m/s}^2$$

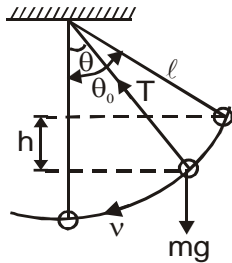
$$5. \quad 1.15 \times 10^8 = \frac{900(10 + a)}{\left(\frac{\pi d^2}{4}\right)}$$

$$\Rightarrow d = \frac{6}{\sqrt{10 \pi}} \text{ cm} = \frac{0.06}{\sqrt{10 \pi}}$$

$$m = \frac{6 \times 10^{-2}}{\sqrt{10 \pi}} \text{ m}$$

$$\text{Ans. } \frac{6 \times 10^{-2}}{\sqrt{10 \pi}} \text{ m}$$

6. The situation is shown in figure.
 (a) From figure $h = \ell (\cos \theta - \cos \theta_0)$
 and $v^2 = 2gh$
 $= 2g\ell (\cos \theta - \cos \theta_0)$ (1)
 Again $T - mg \cos \theta = mv^2 / \ell$ (2)
 Substituting the value of v^2 from eq. (1) in eq. (2)
 we get



$$T - mg \cos \theta = m \{2g\ell (\cos \theta - \cos \theta_0) / \ell \}$$

$$\text{or } T = mg \cos \theta + 2mg (\cos \theta - \cos \theta_0)$$

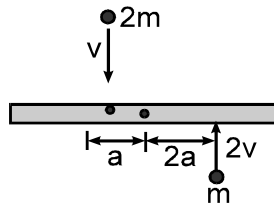
$$\text{or } T = mg (3 \cos \theta - 2 \cos \theta_0)$$

$$\text{or } T = 40g (3 \cos \theta - 2 \cos \theta_0) \text{ newton}$$

Ans. $T = 40 (3 \cos \theta - 2 \cos \theta_0) \text{ kg f.}$

- (b) Let θ_0 be the maximum amplitude. The maximum tension T will be at mean position where $\theta = 0$.
 $\therefore T_{\max} = 40 (3 - 2 \cos \theta_0)$
 But $T_{\max} = 80$
 Solving we get $\theta_0 = 60^\circ$
Ans. $\theta_0 = 60^\circ$

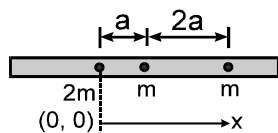
9. (i) Cons. linear momentum



$$-2m \cdot v + 2v \cdot m = 0 = MV_{\text{cm}}$$

$$V_{\text{cm}} = 0$$

(ii) As ball sticks to Rod
 Conserving angular momentum about C
 $2v \cdot m \cdot 2a + 2mva = I\omega$



$$= \left(\frac{8m \cdot 36a^2}{12} + 2m \cdot a^2 + m \cdot 4a^2 \right)$$

$$6mv.a = 30 ma^2.\omega$$

$$\Rightarrow \omega = \frac{v}{5a}$$

$$(iii) KE = \frac{1}{2} I\omega^2 = \frac{1}{2} \cdot 30 ma^2 \times \frac{v^2}{25a^2}$$

$$= \frac{3mv^2}{5}.$$

